Tukey HSD / Tukey–Kramer Post Hoc Analysis

ANOVA result:

	target_variable	group_variable	number_of_groups	total_n	grand_mean	ss_between_groups	df_between_groups	ms_between_groups	ss_within_error	df_within_error	ms_within_error	f_statistic	p_value	eta_squared	omega_squared	alpha	anova_decision_alpha_0_05
1	G3	studytime	4	649	11.90601	465.0778	3	155.0259	6298.189	645	9.764634	15.87627	5.705728e−10	0.06876527	0.06434105	0.05	Reject equal means

	posthoc_note
1	Tukey HSD pairwise results should be interpreted after the omnibus ANOVA context.

Levene/Brown–Forsythe variance context:

	context_test	statistic	p_value	alpha	decision_alpha_0_05
1	Median–centered Levene / Brown–Forsythe context	1.026312	0.3803575	0.05	Equal variance assumption looks reasonable

	interpretation
1	Tukey HSD is an equal–variance post hoc method; use Games–Howell or Tamhane when variances are clearly unequal.

Group summary:

	group	n	mean	standard_deviation	standard_error	variance	minimum	maximum	ci95_low	ci95_high
1	1	212	10.84434	3.218624	0.2210560	10.359541	0	18	10.41107	11.27761
2	2	305	12.09180	3.243125	0.1857008	10.517860	0	19	11.72783	12.45578
3	3	97	13.22680	2.502104	0.2540502	6.260524	8	18	12.72887	13.72474
4	4	35	13.05714	3.038410	0.5135850	9.231933	6	19	12.05052	14.06377

Homogeneous subset letters:

	group	mean	tukey_hsd_homogeneous_subset_letters
3	3	13.22680	A
4	4	13.05714	A
2	2	12.09180	B
1	1	10.84434	C

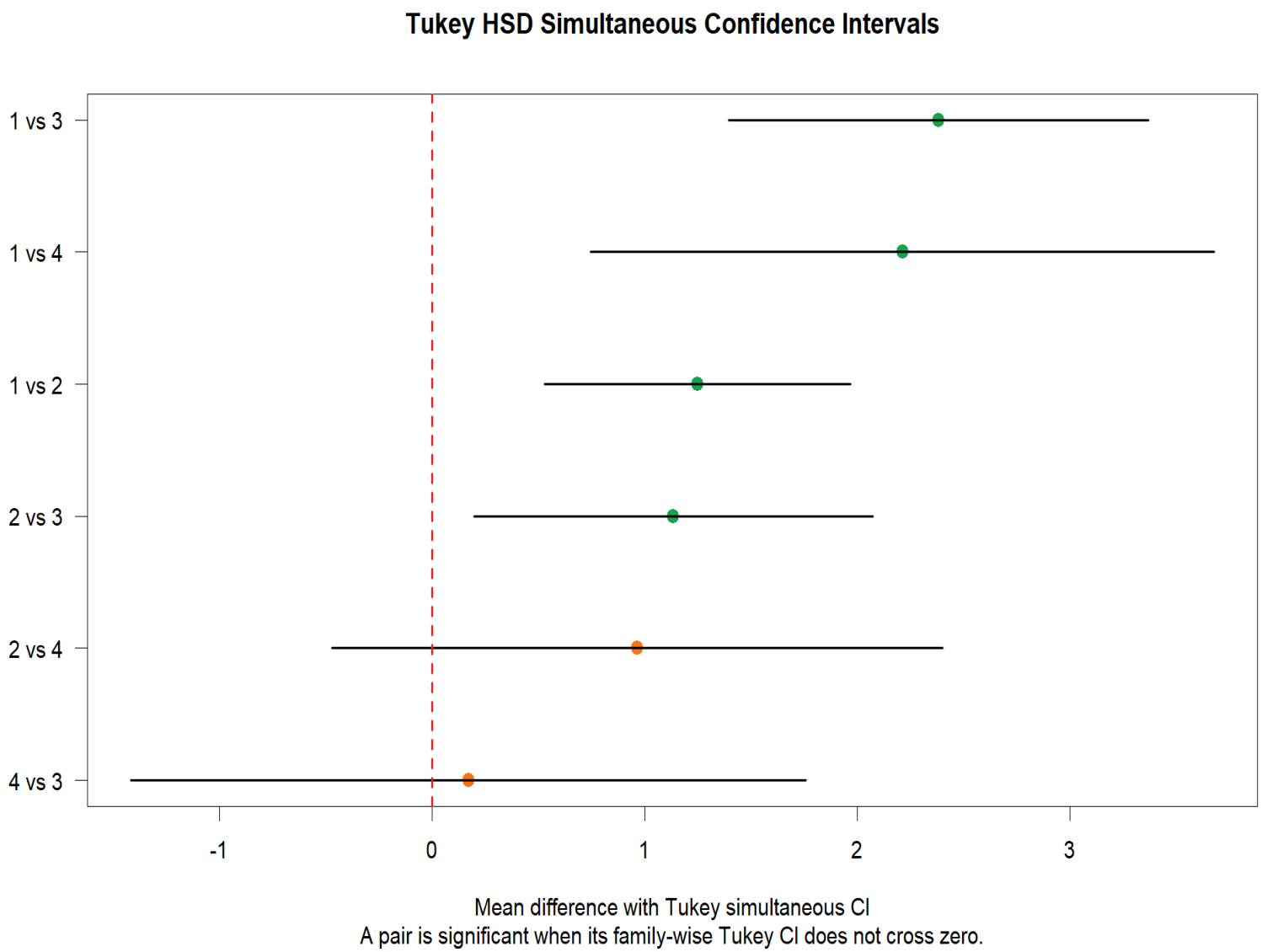
	letter_rule
3	Groups sharing a letter are not significantly different by Tukey HSD at alpha .05.
4	Groups sharing a letter are not significantly different by Tukey HSD at alpha .05.
2	Groups sharing a letter are not significantly different by Tukey HSD at alpha .05.
1	Groups sharing a letter are not significantly different by Tukey HSD at alpha .05.

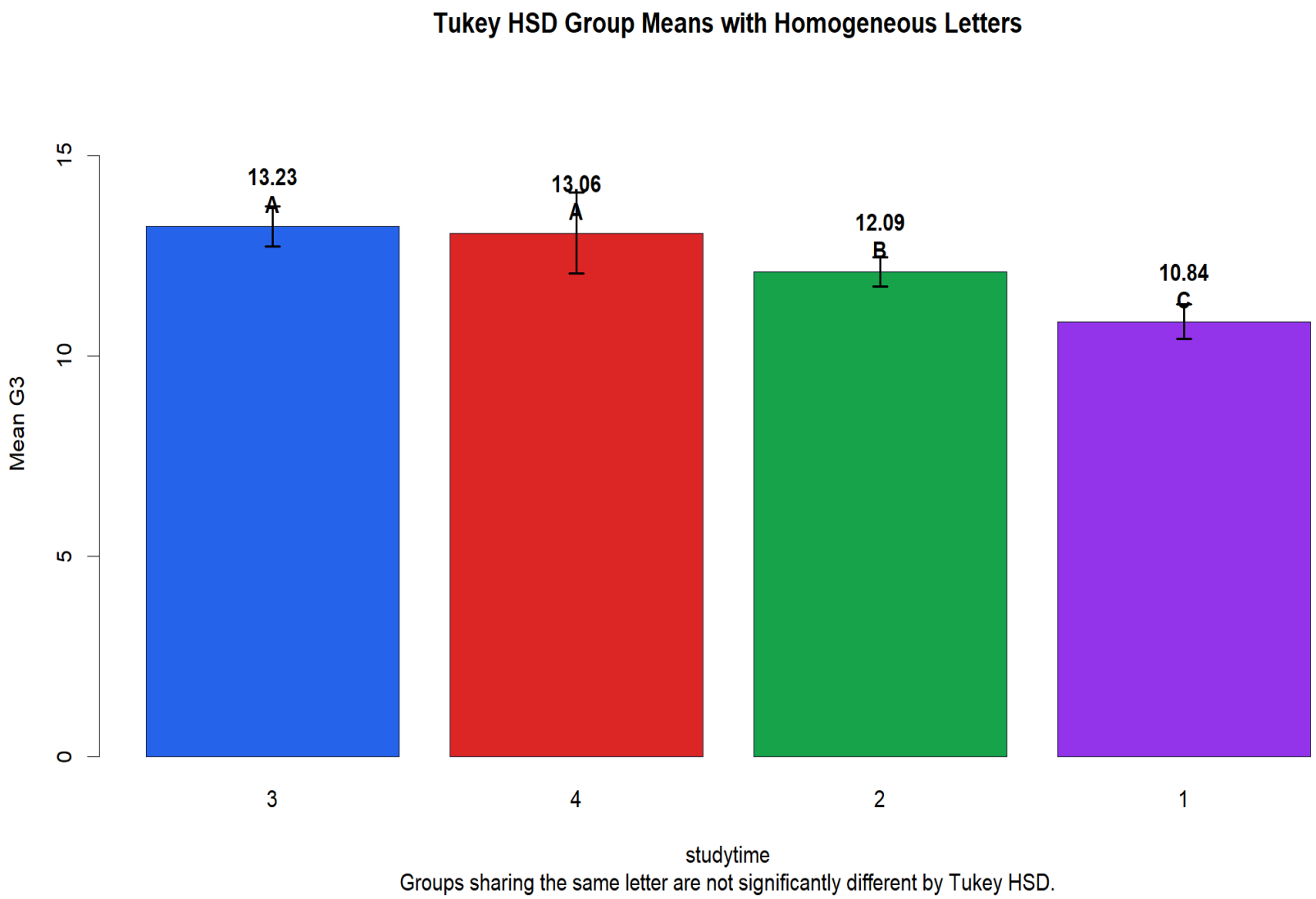
Tukey HSD pairwise comparisons

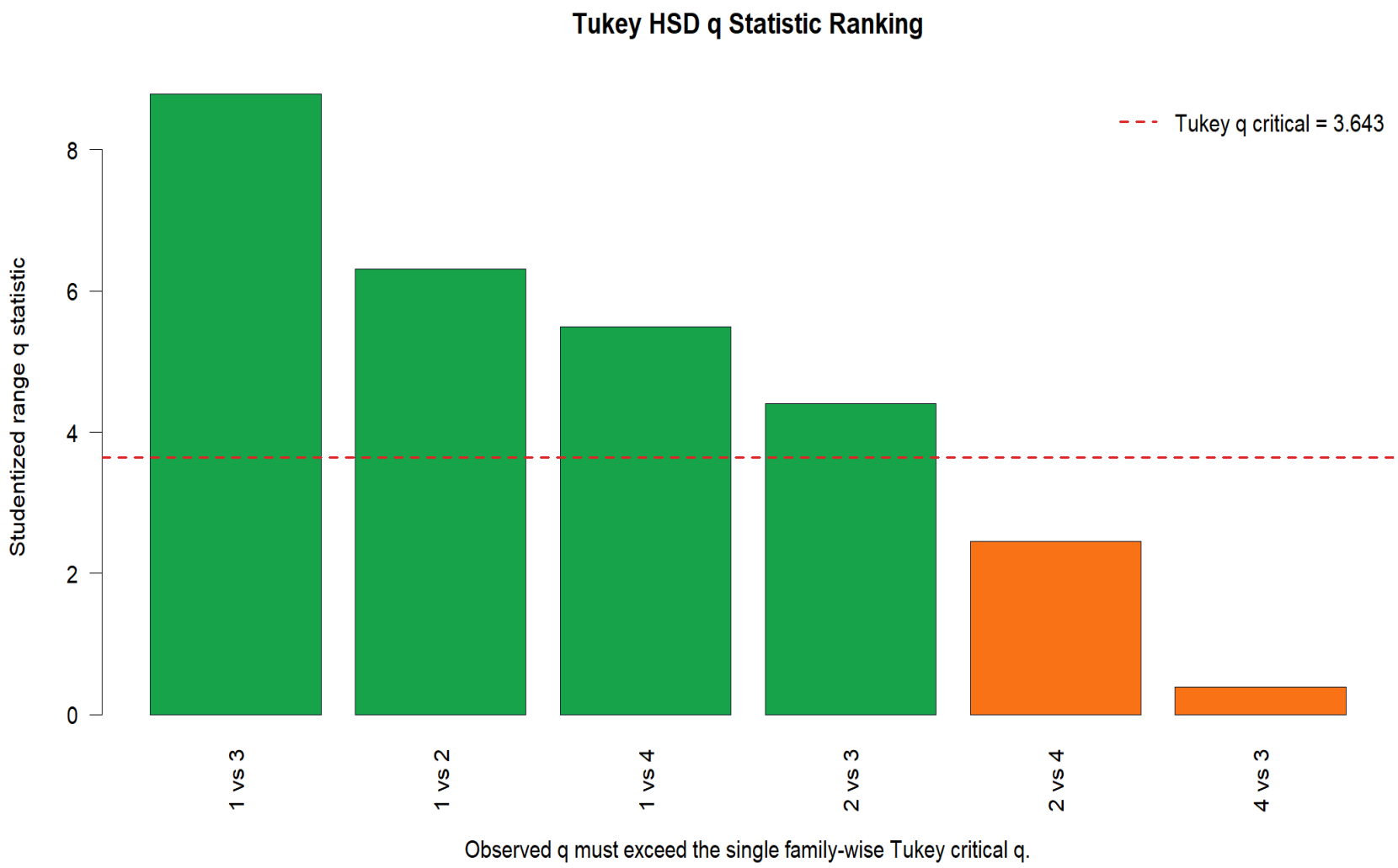
comparison_order	group_1_lower_mean	group_2_higher_mean	mean_1	mean_2	mean_difference_2_minus_1	standard_error_tukey_kramer	q_statistic	q_critical_tukey	hsd_threshold	simultaneous_ci_low	simultaneous_ci_high	df_error	family_number_of_groups	adjusted_p_value_tukey_hsd	decision_alpha_0_05	method_note
1	1	2	10.84434	12.09180	1.2474637	0.1975788	6.313752	3.642648	0.7197101	0.5277536	1.967174	645	4	5.590331e−05	Significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.
2	1	4	10.84434	13.05714	2.2128032	0.4031434	5.488874	3.642648	1.4685092	0.7442940	3.681312	645	4	6.612040e−04	Significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.
3	1	3	10.84434	13.22680	2.3824645	0.2708560	8.796057	3.642648	0.9866329	1.3958316	3.369097	645	4	5.071332e−09	Significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.
4	2	4	12.09180	13.05714	0.9653396	0.3943379	2.448001	3.642648	1.4364340	−0.4710944	2.401774	645	4	3.083885e−01	Not significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.
5	2	3	12.09180	13.22680	1.1350008	0.2575670	4.406624	3.642648	0.9382258	0.1967750	2.073227	645	4	1.031067e−02	Significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.
6	4	3	13.05714	13.22680	0.1696613	0.4356925	0.389406	3.642648	1.5870742	−1.4174129	1.756735	645	4	9.927036e−01	Not significant	Tukey–Kramer HSD uses one family–wide q critical value based on all groups and ANOVA error df.

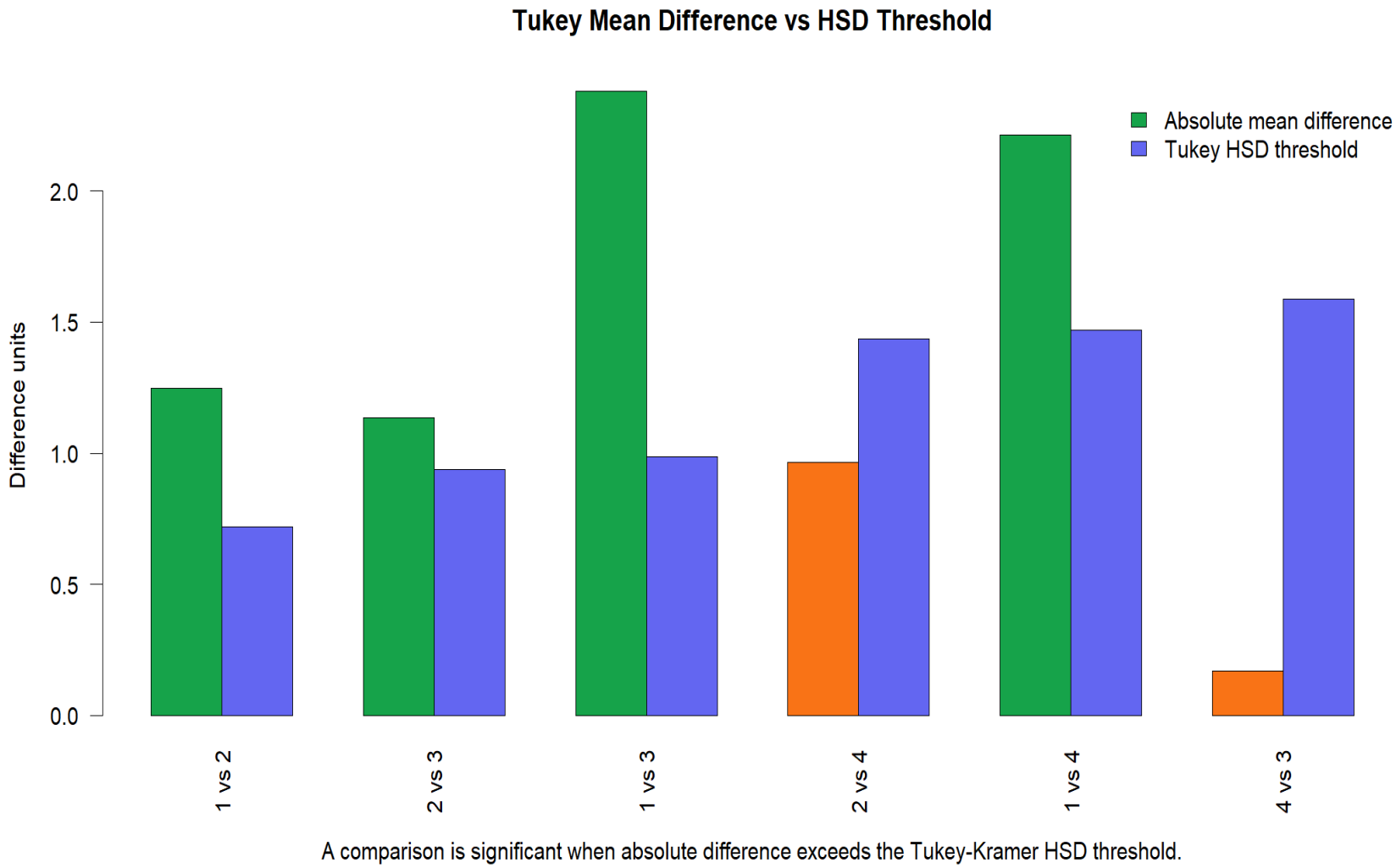
Interpretation summary

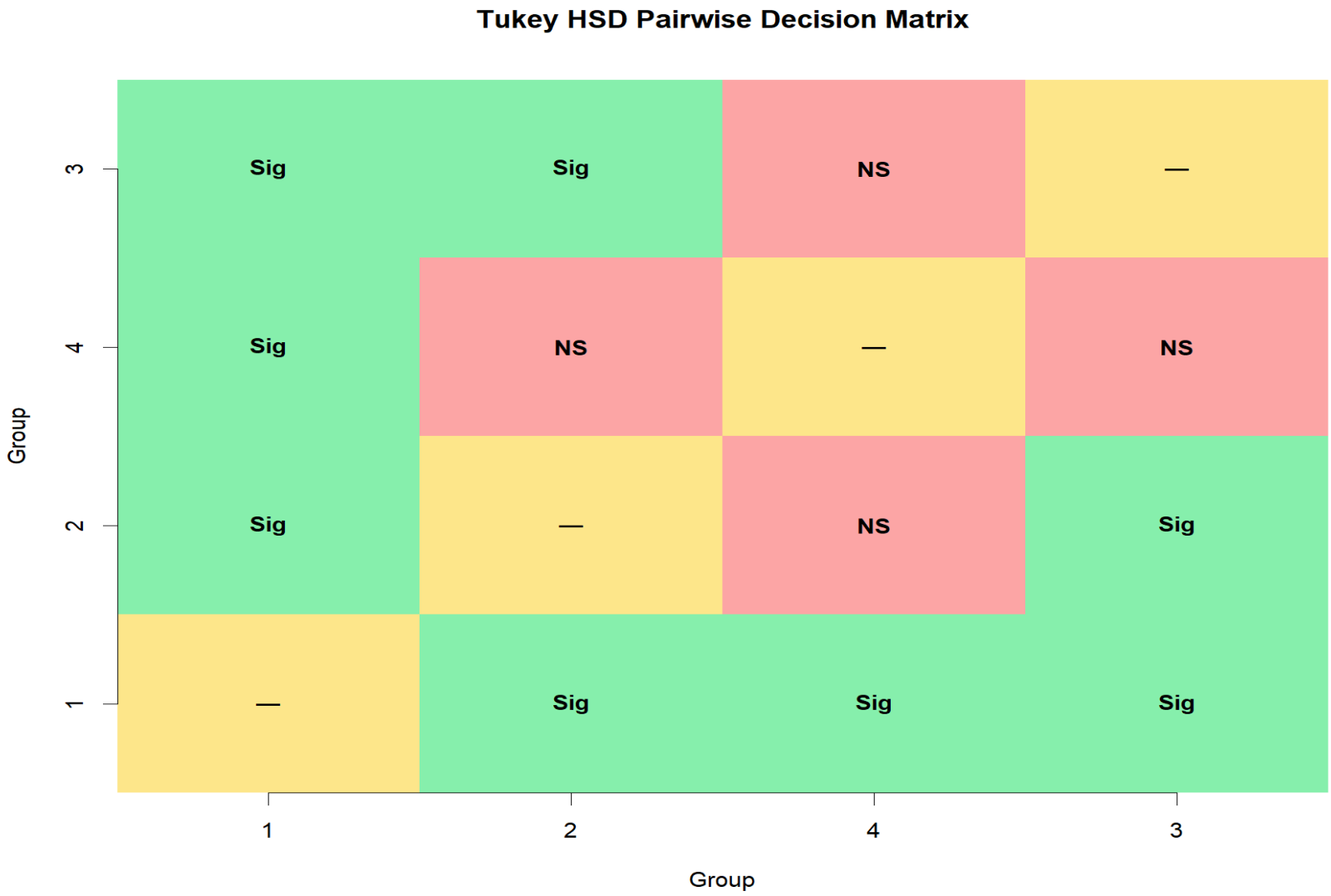
section	finding	interpretation
Method	Tukey HSD / Tukey–Kramer post hoc test	Used when comparing all pairs of group means after one–way ANOVA under an equal–variance context.
Omnibus ANOVA	ANOVA p–value = 0.000000	Proceed with pairwise Tukey interpretation when the overall group mean difference is meaningful.
Variance context	Levene/Brown–Forsythe p–value = 0.380358	Tukey HSD assumes equal variances. If this is violated, compare with Games–Howell or Tamhane's T2.
Pairwise result	4 of 6 Tukey comparisons were significant at alpha .05	Significant pairs have simultaneous confidence intervals that exclude zero and q statistics above the Tukey critical q.
Subsets	3: A; 4: A; 2: B; 1: C	Groups sharing a letter belong to the same Tukey homogeneous subset.

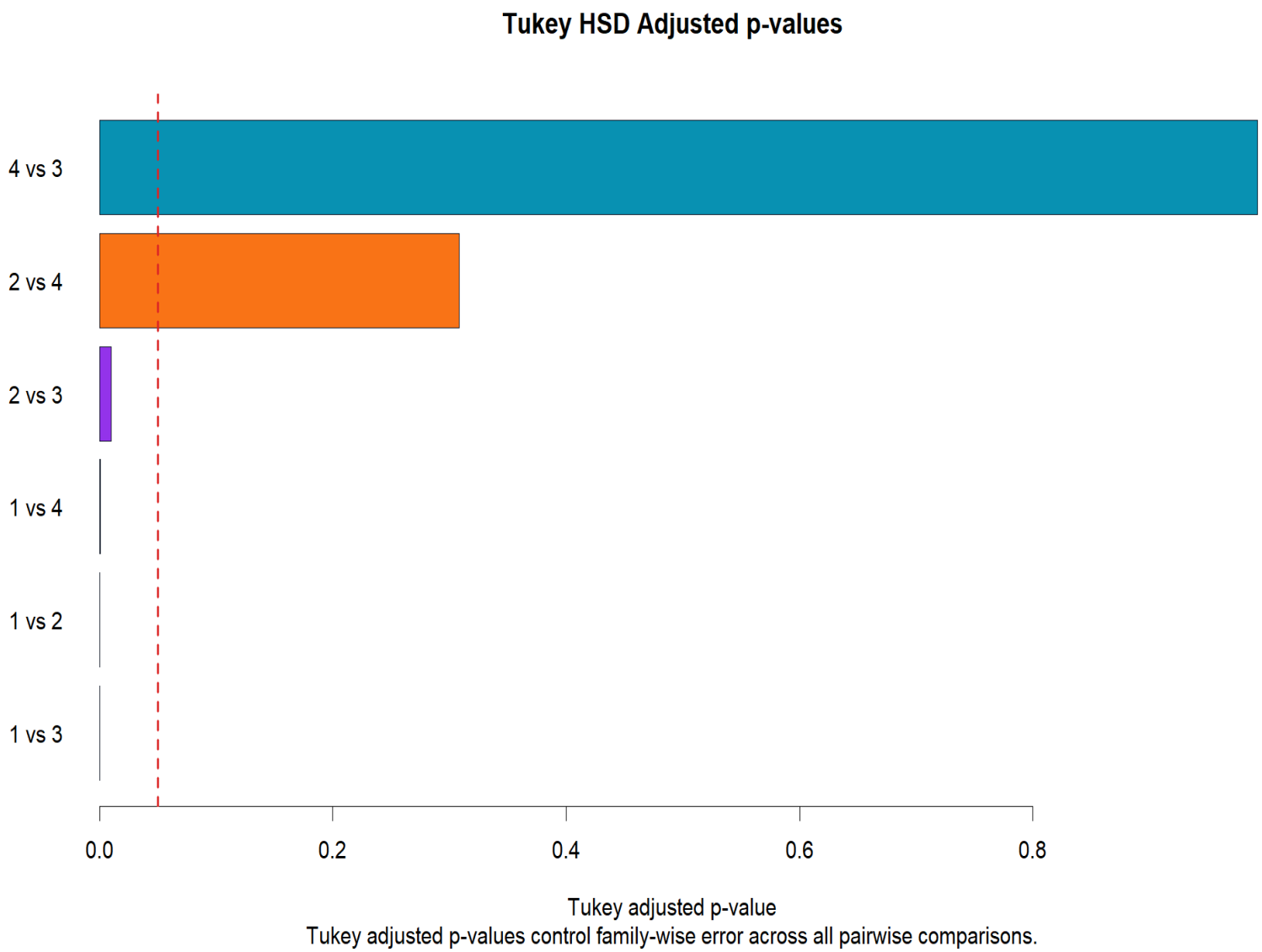

Tukey HSD Simultaneous Confidence Intervals
1 vs 3
1 vs 4
1 vs 2
2 vs 3
2 vs 4
4 vs 3
-1
0
1
2
3
Mean difference with Tukey simultaneous CI
A pair is significant when its family-wise Tukey CI does not cross zero.

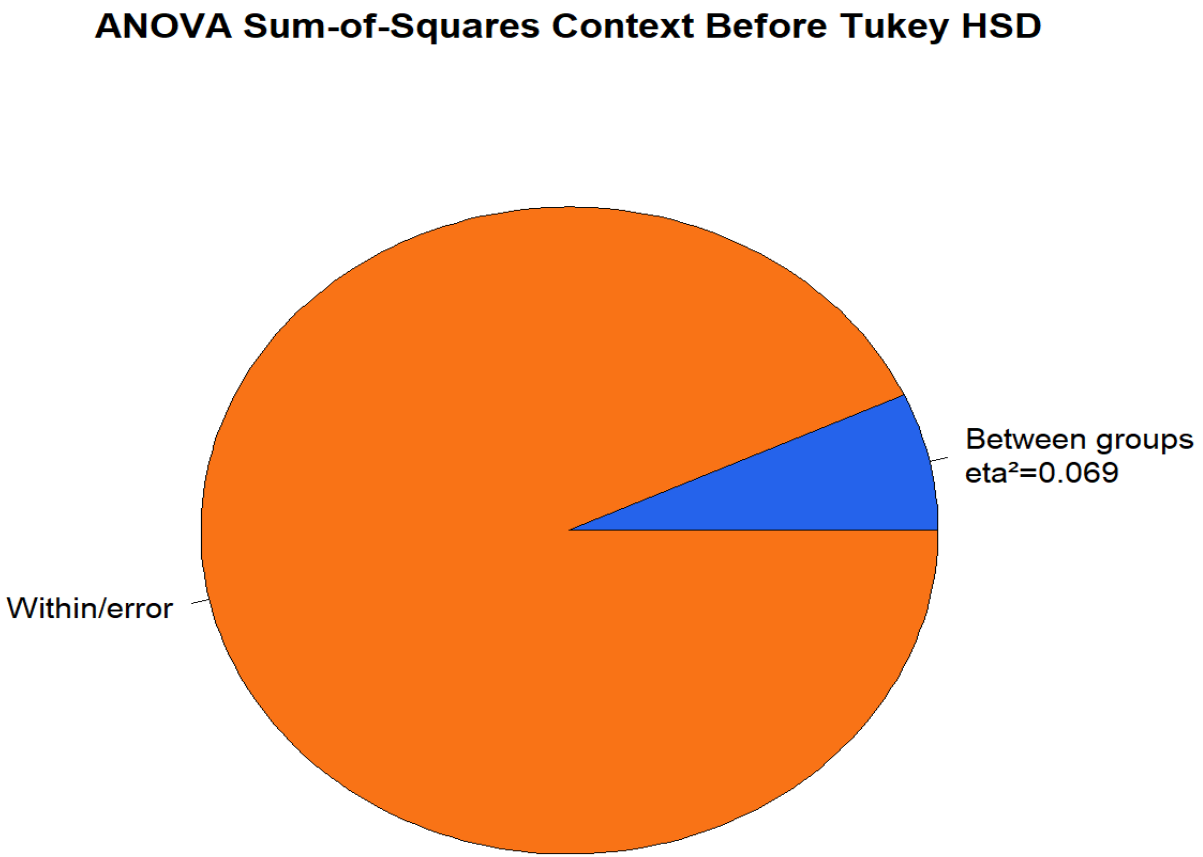

Tukey HSD Group Means with Homogeneous Letters
Mean G3
0
5
10
15
13.23
A
13.06
A
12.09
B
10.84
C
3
4
2
1
studytime
Groups sharing the same letter are not significantly different by Tukey HSD.

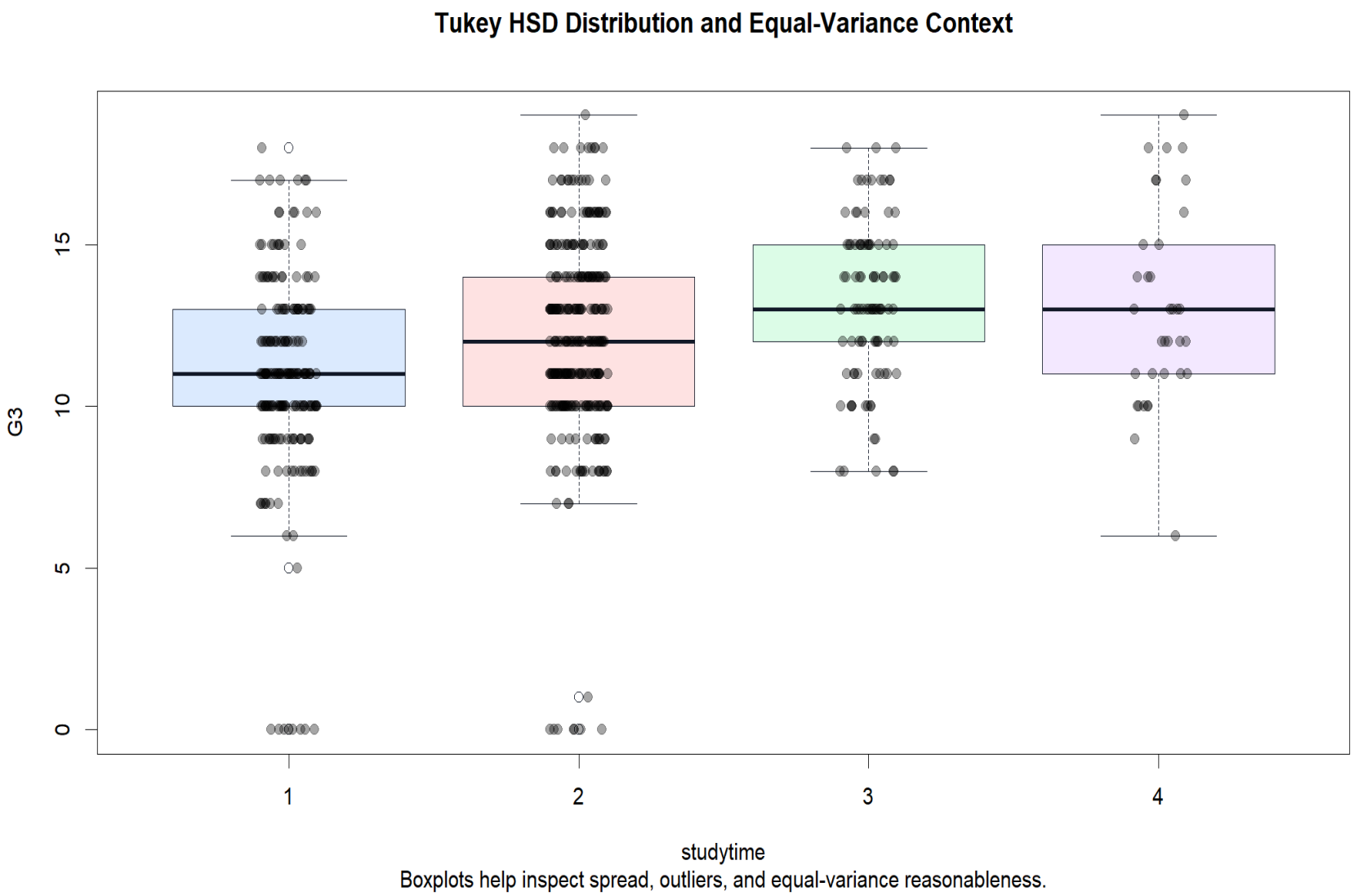

Tukey HSD q Statistic Ranking
Tukey q critical = 3.643
Studentized range q statistic
0
2
4
6
8
1 vs 3
1 vs 2
1 vs 4
2 vs 3
2 vs 4
4 vs 3
Observed q must exceed the single family-wise Tukey critical q.


Tukey Mean Difference vs HSD Threshold
Absolute mean difference
Tukey HSD threshold
Difference units
2.0
1.5
1.0
0.5
0.0
1 vs 2
2 vs 3
1 vs 3
2 vs 4
1 vs 4
4 vs 3
A comparison is significant when absolute difference exceeds the Tukey-Kramer HSD threshold.


Tukey HSD Pairwise Decision Matrix
Sig
Sig
NS
—
Sig
NS
—
NS
Sig
—
NS
Sig
—
Sig
Sig
Sig
3
4
2
1
Group
1
2
4
3
Group


Tukey HSD Adjusted p-values
4 vs 3
2 vs 4
2 vs 3
1 vs 4
1 vs 2
1 vs 3
0.0
0.2
0.4
0.6
0.8
Tukey adjusted p-value
Tukey adjusted p-values control family-wise error across all pairwise comparisons.


ANOVA Sum-of-Squares Context Before Tukey HSD
Between groups
eta²=0.069
Within/error


Tukey HSD Distribution and Equal-Variance Context
G3
0
5
10
15
1
2
3
4
studytime
Boxplots help inspect spread, outliers, and equal-variance reasonableness.